Solution by H. B. LEONARD, B. S.

From $ax^2+by^2+cz^2=0=\alpha ax+b\beta y+c\gamma z$, $a\alpha^2+\beta^2 b+c\lambda^2=1$, we get

$$x^2(a\alpha^2+c\lambda^2)a+2a\alpha b\beta xy+y^2(b\beta^2+c\gamma^2)b=0,$$

$$\frac{x}{y}=-\frac{a\alpha b\beta\pm\gamma\sqrt{(-abc)}}{a(1-b\beta^2)},\quad \frac{x}{z}=-\frac{a\alpha c\lambda\pm\beta\sqrt{(-abc)}}{a(1-c\gamma^2)},$$

$$\frac{y}{z}=-\frac{b\beta c\gamma\pm\alpha\sqrt{(-abc)}}{b(1-a\alpha^2)}.$$

Assuming a, b, c, α, β, γ to be real, then in order that $x:y:z$ may be real, $\sqrt{(-abc)}$ must be real. From $ax^2+by^2+cz^2=0$, it is clear that a, b, c can not all have the same sign and hence we must have one of the quantities a, b, c negative and the other two positive.

184. Proposed by J. A. CALDERHEAD, B.Sc., Professor of Mathematics, Curry University, Pittsburg, Pa.

If m rows, viz., the h_1th, h_2th, ..., h_mth, be transferred so as to become the 1st, 2nd, ..., mth, without altering the relative positions of the remaining rows, and that n columns, viz., the k_1th, k_2th, ..., k_nth, be similarly transformed the determinant thus obtained is the same as the original or differs from it only in sign according as $h_1+h_2+.....+h_m-\frac{1}{2}m(m+1)+k_1+k_2+.....+k_n-\frac{1}{2}n(n+1)$ is odd or even. [*Muir.*]

Solution by G. W. GREENWOOD, B. A. (Oxon), G. B. M. ZERR, A. M., Ph. D., and H. B. LEONARD, B. S.

In transferring the pth row (or column) to the qth row (or column) there are $p-q$ interchanges of adjacent rows (or columns) and therefore $p-q$ changes of sign. Hence, in the given example, there are

$$(h_1-1)+(h_2-2)+.....+(h_m-m)+(k_1-1)+(k_2-2)+.....+(k_n-n),$$
$$i.\ e.,\ h_1+h_2+.....+h_m-\tfrac{1}{2}m(m+1)+k_1+k_2+.....+k_n-\tfrac{1}{2}n(n+1)$$

changes of sign, and the determinant is unaltered in value, or differs only in sign, according as this value is *even* or *odd*; not odd or even as stated.

185. Proposed by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

Without introducing radicals, eliminate x and y from the equations (1) $ax^2+bx+c=0$, (2) $ay^2+by+d=0$, and (3) $ax^2y^2+bxy+e=0$.

I. Solution by H. F. MacNEISH, A. B., Instructor in Mathematics, University High School, Chicago, Ill., G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va., and J. E. SAUNDERS, Hackney, Ohio,

The eliminant of (1) and (3) is

$$\begin{vmatrix} a, & b, & c, & 0 \\ 0, & a, & b, & c \\ ay^2, & by, & e, & 0 \\ 0, & ay^2, & by, & e \end{vmatrix}=0$$

or for $a \neq 0$,

$$(4) \qquad ac^2y^4-b^2cy^3+y^2(b^2c-2ace+b^2e)-b^2ey+ae^2=0.$$

The eliminant of (2) and (4) is

$$\begin{vmatrix} ac^2 & -b^2c & b^2c-2ace+b^2e & -b^2e & ae^2 & 0 \\ 0 & ac^2 & -b^2c & b^2c-2ace+b^2e & -b^2e & ae^2 \\ a & b & d & 0 & 0 & 0 \\ 0 & a & b & d & 0 & 0 \\ 0 & 0 & a & b & d & 0 \\ 0 & 0 & 0 & a & b & d \end{vmatrix}=0,$$

which reduces to

$$\begin{vmatrix} a(c^2d-b^2c+2ace-b^2e)-b^2c(b+c), & b(abc-bcd-c^2d), & -a^2e^2 \\ a & b & d \\ bc(ac+bd), & ac^2d-(b^2cd-2acde+b^2de-a^2e^2), & be(ae+bd) \end{vmatrix}=0.$$

II. Solution by the PROPOSER.

To avoid the introduction of determinants of high order, we proceed thus: Multiply the third equation by a and replace $a^2x^2y^2$ by $ax^2.ay^2$ obtained from the first and second.

$\therefore b(ax+bx+c)y+bdx+ae+cd=0.$

Substituting in the second equation the value of y thus rationally determined, and dropping the factor a (the case $a=0$ being trivial), we obtain a second quadratic for x:

$$(a+b+d)b^2dx^2+(2ade+2cd^2+bcd-abe-b^2e)bx+(ae+cd)^2-b^2ce=0.$$

The eliminant may now be determined in simple form.

186. Proposed by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

Eliminate x and y from the equations (1) $ax^3+bx^2+cx+d=0$, (2) $ay^3+by^2+cy+e=0$, (3) $ax^3y^3+bx^2y^2+cxy+f=0$, the eliminant to be rational in d, e, f.

Solved by H. F. MacNEISH, A. B., Instructor in University High School, Chicago, Ill., and G. B. M. ZERR A. M., Ph. D., Parsons, W. Va.

Using the same method as in No. 185.

GEOMETRY.

203. Additional solutions of problem 203 have been received from *G. W. GREENWOOD*, B.A. (Oxon) Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill., and *J. CHARLES RATHBUN*, A. B., Assistant in Physics, University of Washington.

205. Solutions of problem 205 have also been received from *G. B. M. ZERR*, A.M., Ph.D., Parsons, W. Va., and *G. W. GREENWOOD*, B. A. (Oxon) Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill.